# Definition of A Function

A function from set A to a set B is a subset of $A \times B$ so that for every $x \in A$, there exists exactly one $y \in B$ so that $(x, y) \in f$

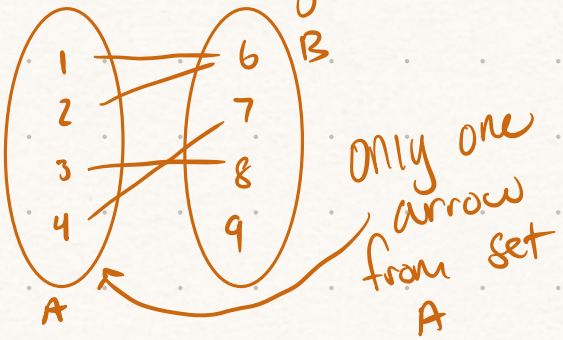
↳ Each x in the domain has exactly one image under f.

$\lfloor \ \rfloor : \mathbb{R} \to \mathbb{Z}$ defined by $\lfloor x \rfloor$ = an integer n needs that $n \leq x < n+1$

$\sqrt{\ } : \mathbb{R}^{\text{non-negative}} \to \mathbb{R}$ defined by $\sqrt{x} = a$ where $a^2 = x$, and $a \geq 0$

$f = g \iff$ f and g have the same domain and for every x in the domain, $f(x) = g(x)$

arrow diagram:




## One to One Functions:

Contrapositive definition:

$f: A \to B$ is one-to-one

$\Updownarrow$

$\forall x, y \in A$, if $f(x) = f(y)$ then $x = y$

$|A| \leq |B| \iff \exists f: A \to B$ so that f is 1-to-1

# On-to function

Everything in the co-domain gets hit.

$f: A \to B$ is onto $\iff \forall y \in B, \exists x \in A, f(x) = y.$ (there exists)

$|A| \geq |B| \iff \exists f: A \to B$ so that $f$ is onto

Let $A = \{1,2,3,4\}$, $B = \{1,2,4,5\}$,

$f = \{(1,1)(2,2)(3,3)(4,4)\}$ is a 1 to 1 function $A \to B$.

$f = \{(1,2)(2,3)(3,4)(4,5)\}$

How many functions are there from $A \to B$?

$5 \times 4 \times 3 \times 2$